The Physical Meaning of m_0c^2 and an Observational Reconstruction of Relativity and Quantum Theory

Real and Measurement Spacetime, OONP, and the Phase–Action Bridge in NNRT

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Abstract

This paper begins with a question concerning the physical meaning of m_0c^2 as it appears in the relativistic energy relation $E^2 = p^2c^2 + m_0^2c^4$. The numerical relations $E = m_0c^2$ and $E = \Delta mc^2$ are supported by precise experiments, but the ontology to which these mathematical quantities are assigned is a separate issue. This question leads directly to a reconsideration of the standard relativistic picture in which the relativistic time and relativistic length introduced by Einstein in 1905, and later geometrized as spacetime after Minkowski, are attributed to the physical world itself. The same issue appears in quantum theory. The relativistic energy–momentum relation connects to relativistic quantum-wave structures and, in the nonrelativistic regime, to Schrödinger mechanics; however, it does not follow uniquely from the mathematics that the wave function, probability, and uncertainty must be interpreted as properties of reality itself. The present paper identifies a common root of these two problems in the identification of mathematically observed quantities with reality itself.

The Nakaza New Relativity Theory (NNRT) places an invariant real spacetime, consisting of universal time and universal space, as its physical basis. It does not, however, assume an absolute rest frame or an ether, and it regards all inertial frames as physically equivalent. In contrast, time, distance, phase, frequency, wavenumber, arrival time, and related quantities constructed through actual measurement are separated as quantities of measurement spacetime. The logical boundary between observational agreement and ontological identification is formulated here as the Observational–Ontological Non-Uniqueness Proposition (OONP). Taking the phase–action correspondence $\Theta = S/\hbar$, $k_\mu^{(\Theta)} = \partial_\mu\Theta$, and $P_\mu^{(\Phi)} = \hbar k_\mu^{(\Theta)}$ as a common bridge, the dispersion structure of a single Fourier mode is organized as relativistic observation, whereas the superposition of multiple modes and Fourier duality are organized as quantum observation. In this framework, $\Delta x\Delta p \geq \hbar/2$ is first interpreted as a bandwidth–localization constraint of a phase-based observational representation, while the Born rule, Hilbert space, and unitary evolution are retained as standard quantum mathematics in the quantum-observational layer. Furthermore, m_0c^2 is reconstructed as the phase-invariant base scale $B_\Phi = \hbar\sqrt{\omega^2 - c^2k^2} = m_0c^2$.

What this paper changes is neither established mathematics nor experimental values. NNRT explicitly rejects the physical definition according to which relativistic time, relativistic length, and relativistic spacetime are properties of real time and real space themselves. It therefore proposes that these concepts be removed from the foundational description of real spacetime and be confined to the mathematics of measurement and observation.

Keywords: NNRT, m_0c^2, OONP, measurement spacetime, invariant real spacetime, phase, action, Fourier transform, uncertainty principle, Born rule, relativity, quantum mechanics

1. Introduction: Two Questions Beginning with m_0c^2

The starting point of this study was a reconsideration of the physical meaning of m_0c^2 in the relativistic energy relation. In the representation $p = 0$, one obtains $E = m_0c^2$, and this quantity is conventionally called the rest energy. It has also been experimentally established with high precision that the relation $E = \Delta mc^2$ holds between a mass difference and emitted energy (Einstein, 1905b; Rainville et al., 2005). The present paper does not question this numerical relation. The question is whether mathematics and experiment uniquely require us to interpret the invariant supplied by the equation as “energy physically existing inside a body at rest.”

Following this question leads to a second problem. Einstein's special relativity introduced relativistic time and relativistic length (Einstein, 1905a), and Minkowski geometrized these structures as four-dimensional spacetime (Minkowski, 1909). In the subsequent standard relativistic picture, Lorentz-type time, length, and metric structure have been treated as structures of real spacetime. On the other hand, the relativistic energy–momentum relation is connected to quantum-wave structures and, in the nonrelativistic regime, to Schrödinger mechanics. Yet it is a separate question why the wave function ψ, Born probability, or the uncertainty relation $\Delta x\Delta p \geq \hbar/2$ must immediately be read as properties of the particle's reality itself. The mathematical success of quantum theory and the ontological meaning assigned to that mathematics can be examined separately.

At first sight, these two questions appear independent. They nevertheless share the same relativistic energy relation,

$$E^2 = p^2c^2 + m_0^2c^4 \quad (1)$$

where E denotes observed energy, p momentum, m_0 invariant mass, and c the speed of light. The numerical validity of Eq. (1) is retained. NNRT asks instead to which layer—the real layer, the phase–action layer, or the observational layer—each quantity belongs, and through which measurement operation it appears. The present paper builds on earlier NNRT discussions separating the roles of the Galilean transformation and Lorentz transformation (Nakaza, 2015, 2023a, 2023b) and extends that observational structure to m_0c^2 and quantum theory.

NNRT places an invariant real spacetime consisting of universal time and universal space as its physical basis and regards all inertial frames as physically equivalent. Real motion is related by the Galilean transformation (GT). By contrast, actual experiments return frequencies, wavenumbers, phase differences, clock readings, arrival times, distances, angles, particle counts, and related quantities; the spacetime description constructed from these quantities is called measurement spacetime. NNRT itself does not claim that real spacetime as such can be directly extracted by an apparatus. Real spacetime belongs to the theoretical foundation, whereas actually measured times and distances are measured quantities obtained through physical standards.

Accordingly, the change that this paper makes to relativity is not merely a change of "assignment." NNRT rejects the definition that makes relativistic time, relativistic length, and relativistic spacetime constituents of real spacetime, and removes them from the physical description of real spacetime. At the same time, it retains the successful mathematics of Lorentz transformations and relativistic metrics as the mathematics of measurement spacetime constructed through electromagnetic waves, clocks, phases, distances, and related operations. This is the essential divergence between the conventional relativistic worldview and NNRT.

With this distinction, the order of physical description also becomes clear. Physical conditions such as sources, relative motion, gravity, and interactions are mapped into phase and measurement responses, resulting in observed quantities such as ν, k, $\Delta\Theta$, clock comparisons, and arrival times. In this paper, the totality of this mapping is denoted by the observation map Φ. Even in a limit without relativistic effects such as relative motion or gravity, measured values remain quantities of measurement spacetime, although their calibration agrees with classical and nonrelativistic spacetime descriptions.

The tasks of this paper can therefore be reduced to two. First, m_0c^2 is reconstructed from the entire relativistic energy relation and the phase structure, and its physical meaning is clarified. Second, beginning from the same phase–action structure, the hierarchy from single-mode to multi-mode descriptions is followed in order to clarify the layer to which uncertainty, Born probability, and Schrödinger structure belong. Before developing these two problems, however, it is necessary to fix the logical boundary of what agreement with observation can determine ontologically. The OONP is therefore introduced in the next chapter.

2. The Starting Point of NNRT: Separation of Observables and Reality (OONP)

The two questions in the preceding chapter share a common issue: whether the mathematics confirmed by observation can be distinguished from the ontological status assigned to that mathematics. OONP is the minimal proposition introduced to make this distinction explicit. Let agreement between a theoretical prediction O_{th} and an experimentally obtained observable O_{exp} be called numerical closure C_{num}, while the condition that the same observational facts uniquely determine the real status of a mathematical object is called ontological closure C_{ont}. Thus numerical closure is $C_{num}: O_{th} \simeq O_{exp}$, whereas ontological closure requires O_{exp} to discriminate independently between distinct ontological interpretations.

C_{num} establishes the fact that "the calculation reproduces observation." C_{ont} requires something stronger: that observation uniquely determine what the mathematical object used in the calculation is in the real world. These are not the same. The problem that empirical equivalence does not uniquely determine ontology has been discussed in the philosophy of relativity, for example in comparisons with Lorentz-type theories (Acuña, 2014) and in the underdetermination between curvature and torsion in GR/TEGR (Mulder & Read, 2024). OONP is the formulation of this general problem within the NNRT separation between observational and real layers.

Let the raw record obtained from an experimental apparatus be D_{exp}, and let the observable constructed from that record be O_{exp}. Let F denote a map from a mathematical structure g internal to the theory to an observational prediction, so that $O_{th} = F[g)$. Here g includes not only a metric but also mathematical structures that generate Lorentz structure, phase four-quantities, and clock comparisons. Thus the measurement side may be represented as $D_{exp} \to O_{exp}$, while the theoretical side is $g \to O_{th} = F[g)$.

Suppose that the same observable O_{th} is produced whether the mathematical structure is interpreted as a structure of real spacetime, g_{real}, or as the structure of measurement spacetime reconstructed from observation, g_{obs}. Then

$$F[g_{real}) = F[g_{obs}) = O_{th} \simeq O_{exp} \not\Rightarrow g_{obs} = g_{real}.$$

The non-implication expresses that agreement of observational predictions does not establish identity between g_{obs} and g_{real}. This is not a mathematical deficiency. It arises because the quantity directly returned by experiment and the ontological name assigned to a theoretical object belong to different logical layers.

This is called the Observational–Ontological Non-Uniqueness Proposition (OONP). OONP is not a proposition that mathematically disproves any particular ontology of real spacetime. It fixes a logical boundary: agreement with observations alone does not uniquely determine the ontological assignment of the mathematical object generating those observations. Consequently, if interpreting a metric as real spacetime and interpreting it as the structure of measurement spacetime give observationally identical predictions, numerical agreement by itself cannot select between the two.

On this non-uniqueness, NNRT places concrete physical axioms. In the real spacetime posited by the theory, universal time and universal space are assumed, all inertial frames are physically equivalent, and real motion is related by the GT. Lorentz-type structures and relativistic metrics are assigned instead to observational structures mediated by phase, electromagnetic waves, clocks, and distances. An important point is that real spacetime itself is not regarded as a directly observed object. NNRT defines it as a physical basis and assigns experimentally obtained time, distance, phase, and related quantities to measurement spacetime.

The two questions concerning m_0c^2 and the quantum state can now be treated under the same principle. The next step is to examine the internal structure of the relativistic energy relation itself. We first extract its velocity dependence as a hyperbolic structure and then proceed to the corresponding phase rotation.

3. Hyperbolic Decomposition of the Energy Relation

To express the velocity-dependent structure of Eq. (1) in hyperbolic form, define $\beta \equiv v/c$ and $\gamma \equiv (1-\beta^2)^{-1/2}$. Then γ and $\gamma\beta$ satisfy

$$\gamma^2 - (\gamma\beta)^2 = 1, \beta = \frac{v}{c} \quad (2)$$

Comparing Eq. (2) with $cosh^2\eta - sinh^2\eta = 1$ defines the rapidity η through

$$\gamma = cosh\eta, \gamma\beta = sinh\eta \quad (3)$$

Using $cosh\eta \pm sinh\eta = e^{\pm\eta}$, the same velocity dependence may be written as

$$\gamma \pm \gamma\beta = e^{\pm\eta} \quad (4)$$

Introducing the half-rapidity variables $a = cosh(\eta/2)$ and $b = sinh(\eta/2)$ gives

$$\gamma = a^2 + b^2, \gamma\beta = 2ab, a^2 - b^2 = 1 \quad (5)$$

To collect the half-rapidity representation into a two-component vector, introduce the real two-component quantity

$$\Psi_R(\eta) = \begin{pmatrix} cosh(\eta/2) \\ sinh(\eta/2) \end{pmatrix} \quad (6)$$

If a generator K satisfying $K^2 = +I$ is introduced, the hyperbolic development is expressed by the exponential map

$$K^2 = +I, e^{\eta K} = Icosh\eta + Ksinh\eta \quad (7)$$

The hyperbolic structure itself is standard mathematics. Its significance here is that it allows comparison, in the same two-component and generator language, with the phase-rotation generator $J^2 = -I$ introduced below. The relativistic velocity dependence has thus been organized by a hyperbolic generator satisfying $K^2 = +I$. NNRT, however, requires not only a structure on the motion side but also the generative structure on the phase side that mediates observation. The next chapter therefore introduces phase rotation and its relation to action through a generator satisfying $J^2 = -I$.

4. Phase–Action Correspondence and Complex Structure

A complex phase $e^{i\Theta}$ can be represented equivalently by the unit circle $S^1 \subset R^2$ in a real two-dimensional plane. Hence two real components provide the minimal representation of a single continuous phase degree of freedom. This is the reason for adopting a two-dimensional representation. Let the phase be represented by a real two-component vector $X = (X_1, X_2)^T$ and suppose a continuous linear evolution $dX/d\theta = AX$ preserves the norm X^TX. Norm preservation for every X requires $A^T = -A$, and in two dimensions this yields, up to normalization, a unique antisymmetric generator J:

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, J^2 = -I \quad (8)$$

The matrix J in Eq. (8) is the generator of rotations in the two-dimensional phase plane; $J^2 = -I$ gives its algebraic distinction from the hyperbolic generator $K^2 = +I$. Exponentiation of J gives the continuous phase rotation

$$e^{J\Theta} = Icos\Theta + Jsin\Theta \quad (9)$$

In the complex representation, J corresponds to the imaginary unit i. We now adopt the standard phase–action correspondence between the action S and the dimensionless phase Θ (de Broglie, 1924; Dirac, 1930):

$$\Theta = \frac{S}{\hbar} \quad (10)$$

If a real amplitude R is attached to the phase Θ, the same rotational structure takes the usual complex-amplitude form

$$\psi = Rexp\left(\frac{iS}{\hbar}\right) \quad (11)$$

The relation $\Theta = S/\hbar$ is a known quantum and Hamilton–Jacobi structure. NNRT uses it as a common bridge between the real layer and the observational layer. The construction specific to this paper is to place this known correspondence within the NNRT real/observational separation and use it as a common bridge between the relativistic single-mode structure and the quantum multi-mode structure.

Once phase and action are related by $\Theta = S/\hbar$, phase is no longer merely the angular variable of an oscillation; it carries physical content as a dimensionless action. Extending this correspondence locally over spacetime places frequency and wavenumber on the same bridge as energy and momentum. The next chapter therefore introduces four-dimensional quantities from the phase gradient.

5. Phase Four-Quantity and Action Four-Quantity

Extend the phase–action correspondence to local quantities on spacetime. Define the spacetime gradient of the phase field $\Theta(x)$ as the phase four-wavenumber $k_\mu^{(\Theta)}$:

$$k_\mu^{(\Theta)} = \partial_\mu \Theta \ (12)$$

Since $S = \hbar\Theta$, its gradient directly connects the action four-quantity with the phase four-wavenumber:

$$P_\mu^{(\Phi)} \equiv \partial_\mu S = \hbar k_\mu^{(\Theta)} \ (13)$$

Separating temporal and spatial components in an inertial-frame representation gives

$$E_\Phi = \hbar\omega, p_\Phi = \hbar k \ (14)$$

The label Φ is used to distinguish these as phase/observational quantities. Equation (13) is the central bridge of the present paper. NNRT does not, however, immediately identify $P_\mu^{(\Phi)}$ with real momentum. Within the scope of this paper, if the low-velocity momentum of GT real mechanics is written as $p_{real} = m_0 v_{GT}$, it is ontologically distinguished from the phase/observational quantity p_Φ:

$$p_{real} = m_0 v_{GT} \not\equiv p_\Phi \ (15)$$

The two are connected through low-velocity and experimental correspondence conditions, but they are not taken to be ontologically identical objects. This separation prevents the NNRT unification of relativity and quantum theory from becoming a "Lorentzization" of real motion.

Even after the phase four-quantity $P_\mu^{(\Phi)}$ has been introduced, its relation to inertial mass in the real layer is not yet fixed. The next chapter determines the correspondence between the base phase scale and the inertial scale through a matching condition.

6. Connection to the First Question: Mass–Phase Base Scale and Inertial Mass

We now return to the first question: the meaning of m_0c^2. Rather than identifying the phase four-quantity and the real inertial quantity from the outset, we examine under what condition the two are connected to the same numerical scale. In what follows, the metric signature is taken to be $(+,-,-,-)$. For a free phase mode, adopt a Lorentz-type phase-invariant structure and denote its nonzero invariant by κ_0:

$$k_\mu k^\mu = \kappa_0^2 \ (16)$$

When a rest representation exists, one may write $\kappa_0 = \omega_0/c$, so that

$$\omega^2 = c^2k^2 + \omega_0^2 \ (17)$$

At this stage ω_0 is not defined from mass. On the phase-observation side, take $\omega(v) = \gamma\omega_0$. Its low-velocity expansion gives

$$\hbar[\omega(v) - \omega_0) = \frac{\hbar\omega_0}{2c^2}v^2 + O\left(\frac{v^4}{c^4}\right) \ (18)$$

On the other hand, write the low-velocity inertial response of GT real mechanics as

$$\Delta H_{real} = \frac{1}{2}m_0v^2 + O(v^4) \ (19)$$

If the correspondence of measured energy differences in the low-velocity limit is imposed as a matching condition, one obtains

$$E_{\Phi 0} \equiv \hbar\omega_0 = m_0c^2 \ (20)$$

This matching provides the first answer to the first question. NNRT does not define the meaning of m_0c^2 from the special representation $p = 0$. Instead, it first reads the phase/observational invariant combination $\hbar^2(\omega^2 - c^2k^2)$ as a whole and takes its positive square root as the phase-invariant base scale. The condition $k = 0$ is only a special representation in which that base scale appears as $\hbar\omega_0$. Calibration against the low-velocity GT inertial response then connects the same scale numerically to m_0c^2.

The relation $\omega_0 = m_0\, c^2/\hbar$ itself is not unique to NNRT. It is known from de Broglie's matter-wave concept through modern discussions of the Compton frequency and matter-wave clocks (de Broglie, 1924; Lan et al., 2013). NNRT does not claim the discovery of this relation. Its claim is that ω_0 should not be presupposed to be a clock measuring real time itself; rather, it is positioned as a quantity connecting the inertial scale of the real layer with the invariant scale of the phase/observational layer.

Once the numerical correspondence between $\hbar\omega_0$ and m_0c^2 is fixed by the low-velocity matching, the invariant of the phase/observational layer can be connected to the mass scale. The relativistic dispersion relation satisfied by each single mode can then be written in closed form. This single-mode branch is made explicit in the next chapter.

7. The Relativistic Single-Mode Branch

Combining Eqs. (13), (16), and (20) reconstructs the mass-shell form for the phase-observational four-quantity:

$$P_\mu^{(\Phi)} P_{(\Phi)}^\mu = m_0^2 c^2 \ (21)$$

Decomposing the Minkowski norm in Eq. (21) into the temporal component E_Φ/c and the spatial component p_Φ gives the usual energy–momentum form,

$$E_\Phi^2 = p_\Phi^2 c^2 + m_0^2 c^4 \ (22)$$

or, in wave/phase variables,

$$(\hbar\omega)^2 - c^2(\hbar k)^2 = m_0^2 c^4 \ (23)$$

$$\omega^2 - c^2 k^2 = \left(\frac{m_0 c^2}{\hbar}\right)^2 \ (24)$$

Equation (24) combines the wave/phase scale, expressed in frequency and wavenumber, with the inertial mass scale in one invariant relation. In the limit $m_0 = 0$, it reduces to $\omega^2 = c^2k^2$ and has the same form as the null dispersion relation for a vacuum electromagnetic wave. For $m_0 > 0$, a base-phase-bearing branch appears. This does not mean that the massive branch has been derived from Maxwell's equations themselves.

The single-mode dispersion relation includes the null branch at $m_0 = 0$. The next issue is where this phase structure connects to actual measurement. Its clearest physical implementation is electromagnetic-wave measurement, and the next chapter therefore examines the relation among phase, frequency, wavenumber, and energy flow inherent in the Maxwell field.

8. The Position of Maxwell Structure and Electromagnetic-Wave Measurement

A plane-wave solution of the vacuum Maxwell equations has phase $\Theta = \omega t - k \cdot x$ and dispersion relation $\omega^2 = c^2k^2$ (Maxwell, 1865). The electromagnetic field also carries an energy density and a Poynting flux. If the energy density of the vacuum electromagnetic field is denoted by u_{EM} and the Poynting vector by S_P, then

$$u_{EM} = \frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2 \ (25)$$

$$S_P = \frac{1}{\mu_0} E \times B \ (26)$$

No simple addition of classical mechanical energy and Maxwell-field energy is intended here. The relativistic energy–momentum relation by itself does not uniquely separate which term is Maxwell-field energy and which is mechanical energy. The proposal of the present paper is a layer-coupling interpretation: classical inertial structure and wave/phase observational structure are coupled within the same observed energy relation.

In NNRT, electromagnetic-wave measurement is a paradigmatic implementation of relativistic observation. This does not reduce every quantum measurement literally to an electromagnetic-wave measurement. Rather, the phase, wave, and spectral-decomposition structures common to quantum observation are treated more generally as phase-mediated measurement.

The Maxwell structure makes clear how phase quantities connect to readings returned by experimental apparatus. A single plane wave, however, cannot represent a signal or object localized in a finite region.

Once localization is required in the observational representation, a multi-mode superposition becomes necessary. The next chapter therefore examines the transition from a single mode to a wave packet.

9. From Dispersion Relation to Wave Packet: The Difference between Single Mode and Localization

The appearance of a dispersion relation $\omega = \omega(k)$ is decisive for the unified structure developed here. A dispersion relation alone, however, does not immediately produce a wave packet. Even a plane wave $exp\{i[kx - \omega(k)t]\}$ with a single value of k satisfies the dispersion relation. A wave packet becomes necessary when localization is required in the observational representation. To represent a signal or object localized in a finite region, many phase modes with different values of k must be superposed:

$$\psi(x,t) = \int A(k)exp\{i[kx - \omega(k)t]\}dk \quad (27)$$

The logical structure adopted in this paper is therefore

$$\text{dispersion} + \text{localization} + \text{superposition} \rightarrow \text{wave packet} \quad (28)$$

Here $A(k)$ is the complex amplitude of each wavenumber component, and its magnitude and phase determine the form of the wave packet. When the spectrum is concentrated near a central wavenumber k_0, a Taylor expansion of $\omega(k)$ gives

$$\omega(k) = \omega(k_0) + (k - k_0)\omega'(k_0) + \frac{1}{2}(k - k_0)^2\omega''(k_0) + \cdots \quad (29)$$

The coefficient $\omega'(k_0)$ of the first-order term gives the translational velocity of the packet envelope. The group velocity v_g is therefore defined as

$$v_g = \frac{d\omega}{dk} \quad (30)$$

The second derivative $d^2\omega/dk^2$ governs the differences in group velocity among wavenumber components and therefore the spreading of the packet. For the massive/base-phase branch used here, $\omega(k) = \sqrt{c^2k^2 + \omega_0^2}$, and hence

$$v_g = \frac{d\omega}{dk} = \frac{c^2k}{\omega} \quad (31)$$

Using $E_\Phi = \hbar\omega$ and $p_\Phi = \hbar k$ then gives

$$v_g = \frac{p_\Phi c^2}{E_\Phi} \quad (32)$$

This equation shows that the relativistic energy–momentum structure and the group velocity of the wave packet are connected through the same phase variables (ω, k). In NNRT, however, the group velocity is not unconditionally identified with the GT velocity of a real particle; the correspondence between them is treated separately as a problem of observational calibration.

A wave packet is a representation formed by superposing many wavenumber components. This multi-mode character simultaneously brings localization in position and spread in wavenumber into the problem. The next chapter therefore treats the wave packet as a Fourier-transform pair and quantifies the relation between localization and bandwidth.

10. Development of the Second Question: Fourier Duality and Bandwidth–Localization

We now turn to the second question: the meaning of uncertainty and observational representation in quantum theory. A wave packet is a Fourier synthesis, and at a fixed time the position-space representation $\psi(x)$ and the wavenumber-space representation $A(k)$ form a Fourier-transform pair (Fourier, 1822). With symmetric normalization,

$$\psi(x) = \frac{1}{\sqrt{2\pi}}\int A(k)e^{ikx}dk \quad (33)$$

$$A(k) = \frac{1}{\sqrt{2\pi}}\int \psi(x)e^{-ikx}dx \quad (34)$$

Representing a narrow structure in position space requires a broad k bandwidth; conversely, approaching a single value of k broadens the position-space representation. For a normalized wave packet, defining Δx and Δk as the corresponding standard deviations gives

$$\Delta x \Delta k \geq \frac{1}{2} \quad (35)$$

Using the phase–action correspondence $p_\Phi = \hbar k$ gives

$$\Delta x \Delta p_\Phi \geq \frac{\hbar}{2} \quad (36)$$

Here the first answer to the second question appears. NNRT does not immediately transfer this inequality into the ontology that "the real particle itself is fundamentally indeterminate." It first places the inequality in the observational layer as a bandwidth–localization constraint inherent in a phase-mediated observational representation. The numerical form of the standard uncertainty relation is retained.

The Heisenberg uncertainty principle (Heisenberg, 1927) and the general Robertson inequality derived from noncommuting operators in Hilbert space (Robertson, 1929) are standard structures of quantum theory. The Fourier-width route used here is numerically consistent with them, but the starting points of the derivations are different. In particular, a Fourier time–frequency width and the general quantum-mechanical time–energy uncertainty relation are not identical concepts.

Fourier duality therefore imposes an unavoidable constraint on the widths of position and wavenumber representations. What has been obtained so far, however, is an amplitude distribution, not a detection probability itself. To keep this distinction explicit, the next chapter identifies the boundary between a Fourier distribution and Born probability.

11. From Fourier Distribution to Quantum Probability Distribution: Retaining the Born Rule as a Boundary

In multi-mode observation, $A(k)$ gives a spectral amplitude distribution and $\psi(x)$ its Fourier reconstruction. At this stage the relation is

$$\psi(x) \leftrightarrow A(k)\text{: Fourier amplitude distributions} \quad (37)$$

and does not yet mean a probability density. In quantum mechanics, $\left|\psi(x)\right)^2$ in the position representation and, after the appropriate change of variable, $\left|A(k)\right)^2$ in the momentum representation are read as measurement probability densities. Only at this point is the Born rule introduced (Born, 1926):

$$\text{Fourier distribution} \xrightarrow{\text{Born rule}} \text{quantum probability distribution} \quad (38)$$

Thus a Fourier transform does not itself create probability. Multi-mode phase-mediated observation produces a Fourier-type distribution structure, and the standard Born readout maps that distribution into measurement probability. Distinguishing these two stages prevents the identification of a Fourier amplitude distribution with quantum probability.

For example, in two-path interference, if $\psi = \psi_1 + \psi_2$, then $|\psi)^2$ contains a cross term depending on the relative phase:

$$|\psi_1 + \psi_2)^2 = |\psi_1)^2 + |\psi_2)^2 + 2\Re\left(\psi_1\ \psi_2\right) \quad (39)$$

The form of the probability distribution is therefore determined by phase relations, while the principle identifying the squared quantity with the frequency of individual detection outcomes remains the Born rule. At this point, relativistic observation and quantum observation can be viewed together as different hierarchical levels of the same phase–action structure. The next chapter summarizes this hierarchy.

12. Hierarchical Structure of Single-Mode Relativity and Multi-Mode Quantum Observation

The preceding results allow the relation between relativity and quantum theory to be fixed as one minimal chain. Each Fourier mode obeys the dispersion relation $\omega^2 = c^2k^2 + \omega_0^2$, which is the relativistic constraint on the single-mode side. Superposing multiple modes for localization forms a wave packet, introduces Fourier duality, and leads to uncertainty, interference, and distribution structure. In quantum theory, the Born rule reads this distribution as a probability distribution.

$$\text{single-mode} \rightarrow \text{relativistic dispersion} \quad (40)$$

$$\text{multi-mode} \rightarrow \text{wave packet} \rightarrow \text{Fourier duality} \rightarrow \text{uncertainty/distribution} \xrightarrow{\text{Born}} \text{probability} \ (41)$$

According to this structure, relativity and quantum theory are not simply two independent branches splitting to the left and right from a common phase. Relativistic dispersion is the constraint satisfied by each component, whereas quantum Fourier structure is the structure by which the set of components is localized, superposed, and read out. This single-mode/multi-mode correspondence is the center of the unified structure proposed here.

To compare this hierarchy directly with the notation of standard quantum mechanics, it is useful to state explicitly the Fourier normalization and definitions of variance. The next chapter adds no new physical content; it simply confirms that the uncertainty relation takes the conventional standard form.

13. Supplement: Standard Form of the Fourier Representation and Uncertainty Relation

In one dimension, adopt the symmetric Fourier normalization and define the position representation $\psi(x)$ and wavenumber representation $\phi(k)$ as the transform pair

$$\psi(x) = \frac{1}{\sqrt{2\pi}} \int \phi(k) e^{ikx} dk \ (42)$$

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int \psi(x) e^{-ikx} dx \ (43)$$

Under the standard normalization and variance conditions, Fourier duality gives

$$\Delta x \Delta k \geq \frac{1}{2} \ (44)$$

Using $p = \hbar k$ yields

$$\Delta x \Delta p \geq \frac{\hbar}{2} \ (45)$$

NNRT first reads this relation not as the intrinsic indeterminacy of the real state itself, but as the dual constraint between localization and bandwidth in a phase-observational representation. This Fourier route is consistent with the Robertson inequality obtained from the operator commutator, but their mathematical premises are not conflated (Robertson, 1929).

Having confirmed the standard form of the Fourier width, bandwidth–localization constraints can now be distinguished mathematically from quantum probability. The next step is to extend the transition from interference amplitudes to actual measurement probabilities to the general framework of quantum measurement. The Born rule is therefore positioned in the density-operator and POVM formalism in the next chapter.

14. Boundary between Probability Distribution and the Born Rule: Connection to Standard Quantum Theory

The superposition of multiple modes contains interference terms that depend on phase differences. If the complex amplitudes for two paths are ψ_1 and ψ_2, then

$$\psi = \psi_1 + \psi_2 \ (46)$$

$$|\psi)^2 = |\psi_1)^2 + |\psi_2)^2 + 2\Re(\psi_1 \ \psi_2) \ (47)$$

In a general quantum measurement, let the state be represented by the density operator ρ and the POVM element corresponding to outcome a by E_a. The Born rule then takes the trace form (Born, 1926; Dirac, 1930)

$$P(a) = Tr(\rho E_a) \ (48)$$

Equation (48) is a rule that gives a probability $P(a)$ from the pair consisting of the standard quantum state ρ and the measurement operator E_a; it is not a rule contained in a Fourier amplitude itself. NNRT retains this mathematics and does not identify ρ or ψ with the real state itself, but reads them as phase-mediated observational representations. This repositioning does not mean that the Born rule is derived from Maxwell's equations.

There is a broad literature concerning the ontological status of quantum states, including epistemic/ontic models, contextuality, and the PBR theorem (Spekkens, 2005; Harrigan & Spekkens, 2010;

Pusey et al., 2012). The present paper does not claim to evade those no-go results. It is limited to making explicit the NNRT position in which standard quantum mathematics is assigned to the observational layer.

The Born rule determines the probabilistic readout of measurement outcomes. What remains is to show how that observational representation connects to the standard quantum structures of time evolution and spatial translation. The next chapter uses the phase–action correspondence to identify the generators and places nonrelativistic quantum evolution in the same observational layer.

15. The Position of Schrödinger Structure: Quantum Operators and Nonrelativistic Quantum Evolution

Once Fourier duality and Born readout have been separated, the place of the quantum state as an observational representation is fixed. In the standard representation of quantum mechanics, the generators of spatial translation and time evolution are (Schrödinger, 1926; Dirac, 1930)

$$\hat{p} = -i\hbar\nabla, \hat{E} = i\hbar\partial_t \ (49)$$

and the position and momentum operators satisfy the canonical commutation relation

$$[\hat{x}_i, \hat{p}_j) = i\hbar\delta_{ij} \ (50)$$

If one adopts the standard Hilbert space, Born-type transition structure, and continuous reversible time evolution that preserves the transition structure, the standard Wigner–Stone results yield unitary evolution (Wigner, 1931; Stone, 1932). For a free system, imposing translational invariance, isotropy, and the GT low-velocity correspondence gives

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m_0}\nabla^2\psi \ (51)$$

The meaning of this equation in NNRT is not that the Schrödinger equation has been newly derived as the ontology of a real particle itself. Hilbert-space structure and unitary evolution are retained from standard quantum theory, and the corresponding time evolution is positioned as the mathematics of the quantum-observational layer. Thus the successful predictions of quantum theory can be retained without requiring ψ to be identified with reality itself.

At this point, relativistic dispersion, Fourier duality, Born readout, and quantum generators have been connected while retaining their distinct roles. The stage of listing individual equations is complete. The next chapter brings them together as a single structure consisting of the real layer, the phase–action layer, and the observational layer.

16. Unified Structure: Real Layer–Phase–Action Layer–Observational Layer

The most condensed unified picture obtained in this paper can be written as

Real → Phase–Action → {single-mode relativity; multi-mode quantum observation} (52)

The arrows in Eq. (52) do not represent temporal evolution. They represent logical correspondences that organize two types of observational structure from the real-motion layer through the phase–action layer. The relativistic branch appears as the invariant dispersion structure satisfied by each phase component. The quantum branch appears as an observational structure that includes superposition of many phase components, Fourier duality, interference, distributions, and the standard Born readout.

single phase mode → relativistic dispersion (53)

multi-mode phase superposition → Fourier duality → uncertainty/distribution (54)

From this perspective, relativity and quantum theory are not two completely independent branches. They form a hierarchy between “the structure constraining each individual phase mode” and “the structure by which multi-mode combinations are observed.” Although the compact equations summarize the whole relation, the quantities must still be assigned explicitly to their respective layers to prevent real quantities and observational quantities from being conflated again. The next chapter therefore organizes the layers of NNRT in tabular form.

17. Separation of the Real and Observational Layers

The most consistent principle supporting the unity of NNRT is the non-identity of the real layer and the observational layer. The structure developed in this paper can be organized into the four layers shown in Table 1.

Table 1. Four-layer structure in the NNRT Phase–Action Unification

Layer	Principal quantities	Role
Real layer	m_0, x_{real}, p_{real}, GT dynamics	Real motion and inertial structure
Phase–action layer	$S, \Theta, k_\mu^{(\Theta)}, \hbar$	Common bridge connecting reality and observational representation
Relativistic observational layer	$\omega, k, P_\mu^{(\Phi)}$, dispersion	Invariant single-mode structure and relativistic observation
Quantum observational layer	ψ, ρ, Fourier spectrum, POVM	Multi-mode interference, distributions, and quantum readout

Within this layer separation, the mass shell is positioned as an invariant structure of the phase/observational layer, whereas ψ and ρ are positioned as quantum-observational representations. Relativity and quantum theory are unified as different observational representations connected through the common phase–action layer.

The four-layer structure fixes the assignment of the principal quantities of NNRT. The remaining issue is to state what this construction establishes and what it leaves to other theoretical domains. The next chapter therefore defines the scope of the theory and makes explicit the boundary between the central claims of NNRT and surrounding problems.

18. Integration of the Two Questions and the Theoretical Scope of NNRT

The development above places the two questions posed at the beginning of the paper within the same layered structure. The first question, $m_0 c^2$, is positioned as a matching between the inertial scale and the phase-invariant scale. The second question—the status of uncertainty, Born probability, and Schrödinger structure—is positioned as the mathematics of multi-mode observational representation. The NNRT framework closes by placing invariant spacetime and GT motion in the real layer; Θ, S, k_μ, and P_μ in the phase–action layer; and the relativistic single-mode structure and quantum multi-mode structure in the observational layer. OONP supplies the logical boundary that prevents these layers from being conflated again.

Several lines of prior research are relevant to this problem setting. Operational geometry showed how spacetime structure can be constructed from light propagation and free fall (Ehlers, Pirani & Schild, 1972). Dynamical or constructive understandings of the spacetime metric were developed by Brown & Pooley (2001) and Brown (2005), with critical discussion by Norton (2008), while spacetime functionalism has been developed by Knox (2014, 2019) and Lam & Wüthrich (2018). Empirical equivalence in relativistic description has been discussed by Acuña (2014), analogue gravity by Barceló, Liberati & Visser (2011), and the fact that empirical equivalence between GR and TEGR does not uniquely determine the reality of curvature has been examined by Mulder & Read (2024).

NNRT is not a repetition of these approaches. Its specific contribution is to connect the role separation between GT real motion and Lorentz-type observational structure proposed in earlier NNRT work (Nakaza, 2015, 2023a, 2023b) with the distinction between invariant real spacetime and measurement spacetime, the phase–action bridge, and the single-mode/multi-mode hierarchy, thereby organizing relativity and quantum theory within the same observational structure.

There is also a broad literature on the ontology of quantum states, including Spekkens (2005), Harrigan & Spekkens (2010), and Pusey, Barrett & Rudolph (2012). The present paper does not claim to derive anew the Born rule, a general interaction Hamiltonian, or the mechanism of single-event occurrence. What it establishes is the range within which relativistic observation and quantum observation can be placed in the same phase–action structure after the mathematical descriptions of observation are distinguished from the real layer.

To prevent this integration from remaining merely abstract, the next chapter returns to the original question. It explicitly reconstructs m_0c^2 as a phase-invariant base scale and then examines what a direct experiment actually establishes. In this way, the starting point of the paper is recovered without surrendering established experimental facts.

19. Recovery of the First Question: The Physical Meaning of m_0c^2

We now summarize the matching relation obtained above as the final physical answer to the first question. NNRT does not define m_0c^2 from the outset as "energy existing inside a body at rest." The starting point is the full invariant combination appearing in the phase/observational layer. If the momentum used in that layer is denoted by p_Φ, define

$$I_\Phi \equiv E_\Phi^2 - p_\Phi^2c^2 = m_0^2c^4 \quad (55)$$

Here p_Φ is a quantity of the phase/observational layer and is conceptually distinguished from the GT momentum $p_{real} = m_0 v_{GT}$ of the real layer. Using the phase–action relations $E_\Phi = \hbar\omega$ and $p_\Phi = \hbar k$, Eq. (55) becomes

$$I_\Phi = \hbar^2(\omega^2 - c^2k^2) \quad (56)$$

The positive square root of this invariant is defined as the phase-invariant base scale B_Φ:

$$B_\Phi \equiv \sqrt{I_\Phi} = \hbar\sqrt{\omega^2 - c^2k^2} = m_0c^2 \quad (57)$$

The term *phase-invariant base scale* is used because the definition contains no condition of "rest." If $k = 0$ is selected, then $\omega = \omega_0$ and one obtains the special representation $B_\Phi = \hbar\omega_0 = m_0c^2$. The order of definition is important in NNRT. The invariant relation as a whole is placed first, and its positive square root B_Φ is defined as the base scale. The conditions $k = 0$ or $p_\Phi = 0$ are merely particular representations of that base scale.

The Compton-frequency relation $\omega_0 = m_0\, c^2/\hbar$ is itself well known and has been examined experimentally and conceptually as a matter-wave clock (Lan et al., 2013). NNRT does not identify this known relation with an oscillation of real time itself; it interprets it as a correspondence between an inertial scale and a phase/observational scale.

Furthermore, m_0 itself and m_0c^2 are not treated as identical concepts. In the real-motion layer, m_0 is a scale characterizing inertial response. In the phase/observational layer, the same numerical scale can be represented as B_Φ/c^2. This reclassification does not alter established equations. The relation $E^2 - p^2c^2 = m_0^2c^4$, the relations $E = \hbar\omega$ and $p = \hbar k$, and their numerical predictions are all retained.

For a transition from state A to state B, let $m_A > m_B$ and define $\Delta m \equiv m_A - m_B > 0$. The corresponding difference in phase-invariant base scales is

$$\Delta B_\Phi \equiv B_{\Phi,A} - B_{\Phi,B} = c^2(m_A - m_B) = c^2\Delta m \quad (58)$$

This relation alone does not determine a particular transition mechanism. How the frequency of emitted radiation corresponds to the difference in internal phase scale must be supplied by the specific dynamics, including conservation laws and the transition mechanism.

The NNRT answer to the first question is therefore explicit: m_0c^2 is the quantity connecting the inertial scale of the real-motion layer with the invariant base scale of the phase/observational layer. In the base representation,

$$E_{\Phi 0} \equiv m_0c^2 = \hbar\omega_0 \quad (59)$$

This does not change the numerical relation $E = m_0c^2$. It reconstructs its physical meaning by separating it from the prior ontological assumption of "rest energy residing inside a real body" and interpreting it instead as a matching with a phase-invariant scale.

Even after m_0c^2 is reconstructed as a phase-invariant base scale, its connection with established experimental facts must remain intact. The next chapter therefore examines a direct experiment that independently compared a mass difference with emitted energy and confirms that NNRT retains the numerical relation without alteration.

20. What Does a Direct Experiment Establish? $E = \Delta mc^2$

Rainville et al. (2005) independently measured the atomic mass difference Δm before and after neutron capture in silicon and sulfur isotopes and the energy E inferred from the wavelength of the emitted γ rays. Their comparison gave

$$1 - \frac{\Delta mc^2}{E} = (-1.4 \pm 4.4) \times 10^{-7} \quad (60)$$

and therefore supported the numerical relation $E = \Delta mc^2$ to high precision. NNRT accepts this experimental fact without modification. What the experiment directly supplied was the comparison between two measured quantities: the mass difference before and after the reaction and the radiated energy. These quantities and their position in NNRT are summarized in Table 2.

Table 2. Quantities compared in the direct mass-difference/emitted-energy experiment and their position in NNRT

Category	What the experiment provides	Position in NNRT
Mass side	Atomic mass difference Δm associated with the initial and final states	Retained as a difference in inertial scale in the real-motion layer
Radiation side	Emitted energy E inferred from the γ-ray wavelength	Quantity measured through electromagnetic frequency, wavenumber, and phase
Comparison	Precise agreement with $E = \Delta mc^2$	Numerical relation fully retained; physical meaning reconstructed according to the distinction between real spacetime and the observational layer

This experiment illustrates the NNRT reading directly. The correspondence between mass difference and radiated energy is an observational fact. NNRT reads it as a correspondence between a change in the inertial scale of the real layer and a change in the energy scale appearing in the phase/electromagnetic measurement layer. Therefore, the validity of $E = \Delta mc^2$ and the interpretation of m_0c^2 as "substantial energy residing inside a body at rest" are separate issues.

20.1 Returning the Same Principle to Relativistic Spacetime Observation

The principle clarified through m_0c^2 can be returned directly to relativistic spacetime observations. Gravitational redshift records a frequency ratio; the Shapiro delay records an arrival time; light deflection records an angle; interferometers record a phase difference; and clock-comparison experiments record clock readings. It is possible to reconstruct spacetime geometry operationally from free fall and light propagation (Ehlers, Pirani & Schild, 1972). The NNRT question concerns the additional step of identifying that reconstructed quantity with real spacetime itself.

In NNRT, a metric that correctly reproduces these observations is denoted $g_{\mu\nu}^{obs}$ and is used as the metric of measurement spacetime. Likewise, curvature reconstructed from observational data has meaning as measured curvature $R_{\mu\nu\rho\sigma}^{obs}$. However, it does not follow independently from observation that $R_{\mu\nu\rho\sigma}^{obs}$ is identical to the curvature $R_{\mu\nu\rho\sigma}^{real}$ of real spacetime. This point has a direct conceptual connection with recent discussions showing that empirical equivalence between GR and TEGR does not uniquely determine the reality of curvature (Mulder & Read, 2024).

NNRT therefore does not adopt a definition in which relativistic time, relativistic length, relativistic metric, or relativistic curvature are constituents of real spacetime. These are removed from the foundational description of real spacetime and confined to quantities of measurement spacetime constructed through electromagnetic waves, clocks, phases, and distances. From this standpoint, the experimental success of relativity is not lost. What has succeeded is the mathematics relating observables, and that mathematics is retained in NNRT.

The chain beginning with OONP and proceeding through the phase–action structure, the single-mode/multi-mode hierarchy, the reconstruction of m_0c^2, and the correspondence with direct experiment is now complete. The final chapter summarizes the NNRT physical picture that emerges from this chain.

21. Conclusion: From m_0c^2 to a Reconstruction of Relativity and Quantum Theory

This paper began by asking for the physical meaning of m_0c^2. Following that question revealed that the relativistic energy relation and quantum-wave structure share the same phase–action relation and, more fundamentally, that it is necessary to distinguish "mathematics established by observation" from "the ontology assigned to that mathematics." OONP generalizes this distinction.

NNRT places real spacetime as an invariant foundation consisting of universal time and universal space and treats all inertial frames as physically equivalent. Real motion is related by the GT. NNRT does not, however, claim to have directly measured real spacetime itself. Time, distance, frequency, phase, and related quantities obtained in experiment belong to measurement spacetime, while Lorentz-type structures and relativistic metrics describe the observational structure.

The conclusion of this paper concerning relativity is therefore explicit. NNRT rejects the definition that places relativistic time, relativistic length, relativistic spacetime, and relativistic curvature among the fundamental properties of "real time and real space themselves." These quantities are retained as successful observational mathematics, but are removed from the constitutive concepts of real spacetime and confined to the description of measurement spacetime. NNRT therefore rejects neither observational values nor Lorentz-type mathematics; it rejects the physical worldview that identifies them with real spacetime itself.

Within this layer separation, $\Theta = S/\hbar$, $k_\mu^{(\Theta)} = \partial_\mu \Theta$, and $P_\mu^{(\Phi)} = \hbar k_\mu^{(\Theta)}$ form the common bridge connecting relativity and quantum theory. Conceptually, the chain on the relativistic-observation side may be summarized as

$$\{\text{relative motion}, T_{\mu\nu}, \text{interaction}\} \to \Phi \to k_\mu^{(\Theta)} \to \{\nu, k, \Delta\Theta, \text{clock}, \text{distance}\} \to g_{\mu\nu}^{obs} \quad (61)$$

Here $g_{\mu\nu}^{obs}$ is not real spacetime itself but a representation of measurement spacetime that organizes the observables consistently. On the quantum side, the same phase–action bridge gives the hierarchy

$$\text{real state} \to S \to \Theta = S/\hbar \to \{k_\mu^{(n)}\} \to \psi, \rho \to P(a) = Tr(\rho E_a) \quad (62)$$

Each single mode carries the relativistic dispersion relation, while multi-mode superposition produces wave packets, Fourier duality, interference, and bandwidth–localization constraints. The Born rule, Hilbert space, and unitary evolution are retained as standard quantum mathematics in the quantum-observational layer. Consequently, the numerical content of Schrödinger structure and uncertainty relations need not be changed; their ontological assignment can instead be organized within the observational layer.

The first question is also recovered here. The quantity $B_\Phi = \hbar\sqrt{\omega^2 - c^2k^2} = m_0c^2$ is the phase-invariant base scale, and $\hbar\omega_0 = m_0c^2$ at $k = 0$ is one representation of it. The direct experiment of Rainville et al. (2005), which established $E = \Delta mc^2$, is retained without alteration. What NNRT changes is not the numerical relation but the physical meaning assigned to m_0c^2: it is positioned as a connection between an inertial scale and a phase-invariant scale.

In this way, m_0c^2, uncertainty, Born probability, Schrödinger structure, and the relativistic metric are organized not as unrelated puzzles but as different manifestations of one question: where the boundary between reality and observation should be drawn. By retaining established mathematics and experimental facts while re-examining what is directly measured and what is thereby required as physical fact, a common observational structure linking relativity and quantum theory becomes visible. The question that began with m_0c^2 thus leads ultimately to the NNRT worldview in which relativistic spacetime is removed from the concept of reality and relativity and quantum theory are reconstructed as observational mathematics.

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